MATH 141: Quiz 5

Name: Key

Directions:

* Show your thought process (commonly said as "show your work") when solving each problem for full credit. **Remember to fully simplify.**
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

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1. Find the absolute maximum and absolute minimum of the function

$$f(x) = x^3 + 4x^2 + 1, \quad [-2, 2]$$

Use Closed Interval Method. Check EVT before using.

① Closed interval ✓

② Continuous, yes b/c domain $\mathbb{R}$

① Find crit #s and plug them in.

$f'(x) = 3x^2 + 8x$

ⓐ Solve $f'(x) = 0$

$3x^2 + 8x = 0$

$x(3x+8) = 0$

$x = 0, \quad 3x + 8 = 0$

$x = 0, \cancel{-\frac{8}{3}} \longrightarrow f(0) = 0^3 + 4 \cdot 0^2 + 1 = 1$

ignore $-\frac{8}{3}$ not in $[-2, 2]$

ⓑ $f'(x)$ DNE.

N/A since polynomial.

② Check endpoints

$f(-2) = (-2)^3 + 4(-2)^2 + 1 = -8 + 16 + 1 = 9$

$f(2) = 2^3 + 4 \cdot 2^2 + 1 = 8 + 16 + 1 = 25$

③ $f(-2) = 9$ $\quad f(0) = 1 \longrightarrow$

$f(2) = 25$

$\therefore f(x)$ has an absolute max of $f(2) = 25$
absolute min of $f(0) = 1$

Recall definition. If $u = f(x)$ then $du = f'(x)\,dx$

2. If $u = x^2$, find the differential du.

$$du = f'(x)\,dx = \boxed{2x\,dx}$$

3. Consider

$$f(x) = \frac{1}{x}, \quad [-3, -1]$$

Can you invoke the Mean Value Theorem? If not, explain why you cannot.

If you can invoke the MVT, find all values of c which satisfy the conclusion of the MVT.

Yes, you are allowed to. Here's why:

① $f(x) = \frac{1}{x}$ has domain $(-\infty, 0) \cup (0, \infty)$ so certainly continuous on $[-3, -1]$

② $f'(x) = -\frac{1}{x^2}$ has domain $(-\infty, 0) \cup (0, \infty)$ so differentiable on $(-3, -1)$

By MVT there is a $c \in (-3, -1)$ where

$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad \text{where } a = -1,\ b = -3$$

$$-\frac{1}{c^2} = \frac{f(-1) - f(-3)}{-1 - (-3)} = \frac{\frac{3}{3}\cdot\frac{1}{-1} - \frac{1}{-3}}{-1 + 3} = \frac{\frac{3 - 1}{-3}}{2}$$

$$= \frac{-\frac{2}{3}}{2} = -\frac{2}{3} \cdot \frac{1}{2} = -\frac{1}{3}$$

So $c^2(-3) \cdot -\frac{1}{c^2} = -\frac{1}{3} \cdot (-3) \cdot c^2$

$$\pm\sqrt{3} = \sqrt{c^2}$$

$$c = \pm\sqrt{3} \longrightarrow \boxed{c = -\sqrt{3}} \quad \text{since } -\sqrt{3} \in [-3, -1]$$